PHYSICAL REVIEW VOLUME 129, NUMBER 2 15 JANUARY 1963

# Partial and Total Capture Probabilities of Negative Muons by $C^{12}$*

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(Received 23 July 1962; revised manuscript received 8 October 1962)

The capture probability of negative muons by the $C^{12}$ nucleus is calculated as a sum of partial transition rates to the individual levels of $B^{12}$. The calculation is based on an extreme single-particle shell model. The calculation is an application of the general formalism developed by Morita and Fujii for predicting the transition rate between two definite nuclear states, following muon capture. Regarding bound-state captures, the agreement between theory and experiment is good. An exception is the prediction concerning the 1.67-MeV level of $B^{12}$, the calculated transition rate being too large by a factor whose lower limit is about 2. This casts some doubt on the spin and parity assignments currently accepted for this level. The total capture probability is too small by a factor of 2, as is the ratio of unbound-to-bound captures. The discrepancy is accounted for in part by the existence of little-known, highly excited virtual states of $B^{12}$. These conclusions are of limited scope because of the use of the single-particle shell model.

## INTRODUCTION

THE subject of the following investigation is the negative muon capture reaction:

$$\mu^- + C^{12} \rightarrow B^{12} + \nu.$$

A calculation is made to estimate the probability of capture to the individual levels of the final-state nucleus, and the total probability of capture is then considered as the sum of these partial transition rates. Depending upon how much of the rest mass of the muon is transferred to the absorbing proton, $B^{12}$ may be formed in any of its bound or virtual states. In the former case, $B^{12}$ decays through a beta transition to $C^{12}$, either directly or following gamma de-excitation, while in the latter case a neutron is evaporated, leaving $B^{11}$ which is stable. The calculation also permits an estimate to be made of the ratio between these two branches of the capture reaction.

Previous investigations of this reaction have been concerned solely with transitions to the ground state of boron,[1,2] or have been confined to a calculation of the total probability of capture, using either an average model of the nucleus[3] or a closure approximation.[4,2] The relative simplicity of the level scheme of $B^{12}$ and a recent experiment[5] measuring transition rates to individual bound excited states of this nucleus make it worthwhile to attempt a theoretical prediction of the capture rate to each of these levels. Comparison with experiment and the consistency of agreement or disagreement should enable one to draw tentative conclusions concerning the currently accepted spin and parity assignments of the excited states of $B^{12}$.

A general formalism for calculating the transition rate between any two definite nuclear states following muon capture has been presented by Morita and Fujii,[1] and it is this technique which is used in the present calculation. An extension of the treatment of beta decay by Rose,[6,7] it uses a spherical representation for the weak interaction operators and has the advantage that the angular integrations of the matrix elements are considerably simplified.

The interaction Hamiltonian density assumes vector and axial vector terms of the Fermi type, taking into account both the induced pseudoscalar interaction and that due to a conserved vector current. The reality of the coupling constants follows from the assumption of the time reversal invariance of the weak interaction. Accepting an almost purely negative helicity for the neutrino, one finds that the general lepton covariant reduces to the form

$$\bar{\varphi}\Omega(1+\gamma_5)\psi.$$

Finally, the coupling constants used in this calculation have the following values:

$$C_A{}^\beta = -1.21 C_V{}^\beta, \quad C_A{}^\mu = 0.999 C_A{}^\beta,$$
$$C_V{}^\mu = 0.972 C_V{}^\beta, \quad C_P{}^\mu = 8 C_A{}^\mu.$$

Morita and Fujii applied the theory to predict the capture rate of the transition resulting in $B^{12}$ in its ground state. Assuming the error in the nuclear matrix elements to be the same as that which is found when the calculated rate of the subsequent beta decay is compared with experiment, they multiplied their calculated rate of muon capture by this same factor. Their results are well within the limits set by current experimental data.

Since there are no beta transitions from any of the excited bound states of $B^{12}$, the assumption in this paper is that the matrix elements for transitions in muon capture to each of the excited states of the final nucleus are in error by this same constant factor.

An attempt to present the general expressions required for the calculation would merely reproduce information already available in the previously cited

* Work partially supported by the National Science Foundation and the Air Force Office of Scientific Research.
[1] M. Morita and A. Fujii, Phys. Rev. **118**, 606 (1960).
[2] H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959).
[3] J. Tiomno and John A. Wheeler, Rev. Mod. Phys. **21**, 153 (1949)
[4] H. A. Tolhoek, Nucl. Phys. **10**, 606 (1959).
[5] E. J. Maier (private communication).
[6] M. E. Rose and R. K. Osborne, Phys. Rev. **93**, 1315 (1954).
[7] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), Chap. IX.

article of Morita and Fujii.[8] Consequently, the following treatment is confined to a consideration of the level scheme of $B^{12}$, the nuclear model used, the results of the calculation, and the conclusions that may be drawn when predicted capture rates are compared with experiment.

### THE LEVEL SCHEME OF $B^{12}$

Table I contains the data concerning the energy levels of $B^{12}$. The energy of each level and the spin and parity assignments of column A are taken from Ajzenberg-Selove and Lauritsen.[9] The assignments of column B have been made to favor capture to unbound states of boron, while those of column C favor capture to bound states. Thus, two calculations have been made, placing limits upon the expected neutron emission. Arbitrarily, a spin of 2 has been given to the 2.62- and 2.72-MeV levels, and this has been justified by comparison with what little experimental data exists.

### THE NUCLEAR MODEL

A shell model of the nucleus is used in order to calculate transition rates to the individual levels of boron. This has the advantage that the wave function of each level is given the correct spin and parity, so that the forbiddeness of the transition is accounted for. In the case of $C^{12}$, however, the model has certain obvious disadvantages. The eight-particle configuration renders it practically impossible to consider the mixing of states, so that an extreme single-particle model is the only plausible one if partial transition rates are to be calculated. It is also assumed that $jj$-coupling is a reasonable approximation.

In this model, one assumes that successive pairs of equivalent protons or neutrons couple their spins to zero, and completed shells are regarded as comprising an inert core which does not, therefore, appear in the wave function.

Of the remaining particles, all but one are coupled to form an antisymmetric product $W_{j,\tau}{}^{m,\tau_3}$, whose $j$ value is equal to zero or to the total angular momentum of the last odd nucleon. The excess number of protons or neutrons is specified as usual by $\tau_3$. This product is then coupled to the wave function of the last odd nucleon, whose orbital and total angular momenta are determined by least energy considerations and by the spin and parity of the nuclear level which is of interest.

The single-particle wave functions, $\chi_j{}^m\varphi_{1/2}{}^{\tau_3}$, are a product of space-spin and isospin; the space-spin wave function is written in standard notation

$$\chi_j{}^m=\phi_l(r)\textstyle\sum_\mu(l\tfrac{1}{2}j\,|\,m-\mu,\,\mu)Y_l{}^{m-\mu}(\hat{r})\psi_{1/2}{}^\mu. \qquad (1)$$

The $\phi_l$ is the lowest energy oscillator wave function of orbital momentum $l$ and is given by

$$\phi_l=N_lr^l\exp(-ar^2/2), \qquad (2)$$

where the normalization is for a nuclear radius of 2.52 F.

For the ground state of $C^{12}$, $J^\pi=0^+$, and the configuration is $(\pi_{3/2})^4(\nu p_{3/2})^4$. The normalized wave function may be written

$$\Psi_{J,T}{}^{M,T_3}=\Psi_{0,0}{}^{0,0}=(\tfrac{1}{2})^{1/2}\sum_m\sum_{\tau_3=\pm1/2}(\tfrac{3}{2}\,\tfrac{3}{2}\,0\,|\,m,\,-m)$$
$$\times2\tau_3W_{3/2,1/2}{}^{m,\tau_3}(2,3,\cdots,8)\chi_{3/2}{}^{-m}(1)\varphi_{1/2}{}^{-\tau_3}(1). \qquad (3)$$

That this wave function is properly antisymmetric is evident from the fact that it may be written as a single determinant.

With this simple model, one may write a general wave function which may be made to represent any one of the states of $B^{12}$ when particular values are assigned to the orbital and total angular momenta. Thus, if the antisymmetric product wave function of seven nucleons represents the configuration $(\pi p_{3/2})^3(\nu p_{3/2})^4$, one may write

$$\Psi_{J,T}{}^{M,T_3}=\Psi_{J,1}{}^{M,-1}=(\tfrac{1}{8})^{1/2}\sum_P\sum_m(\tfrac{3}{2}jJ\,|\,m,\,M-m)$$
$$\times e_PPW_{3/2,1/2}{}^{m,-1/2}(2,3,\cdots,8)\chi_j{}^{M-m}(1)\varphi_{1/2}{}^{-1/2}(1). \qquad (4)$$

The letter $P$ denotes a permutation obtained by transposing one of the particle indices in $W$ with 1. That the summation over the eight possible permutations produces the desired antisymmetrization is easily verified. Finally, $e_P$ denotes the sign due to the permutation.

### NUMERICAL RESULTS

The partial capture probabilities are listed in Table II. The transition rates listed in column B have been

TABLE I. The energy levels of $B^{12}$.

| Energy of level above ground state (MeV) | Spin and parity A | B | C |
|---|---|---|---|
| 0.00 | $1^+$ | $1^+$ | $1^+$ |
| 0.95 | $(2^+, 3^+)$ | $3^+$ | $2^+$ |
| 1.67 | $1^-, 2^-$ | $2^-$ | $1^-$ |
| 2.62 | | $2^+$ | $2^-$ |
| 2.72[a] | | $2^+$ | $2^-$ |
| 3.39 | $3^+$ | $3^+$ | $3^+$ |
| 3.75 | $2^+$ | $2^+$ | $2^+$ |
| 4.30 | $1^-$ | $1^-$ | $1^-$ |
| 4.51 | $3^-$ | $3^-$ | $3^-$ |
| 5.00 | 1 | $1^+$ | $1^-$ |
| 5.61 | 2 | $2^-$ | $2^+$ |
| 5.72 | 3 | $3^+$ | $3^-$ |

[a] The levels 0.00–2.72 are bound states.

[8] To include even the final expression for the transition rate would be of little value, since explanation of the included terms would involve one in an ever increasing reproduction of the formulas and tables of Morita and Fujii. The authors feel justified in this omission since Morita and Fujii have tabulated these expressions for the very purpose of facilitating calculation.

[9] F. Ajzenberg-Selove and T. Lauritsen, Ann. Rev. Nucl. Sci. 10, 416 (1960).

TABLE II. Calculated transition rates to the individual levels of boron in the reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$.

| Energy of level above ground state (MeV) | Transition rate (sec$^{-1}$) B | C |
|---|---|---|
| 0.00 | 7290 | 7290 |
| 0.95 | 66 | 63 |
| 1.67 | 1093 | 3353 |
| 2.62 | 62 | 987 |
| 2.72 | 61 | 983 |
| 3.39 | 62 | 62 |
| 3.75 | 57 | 57 |
| 4.30 | 3026 | 3026 |
| 4.51 | 1 | 1 |
| 5.00 | 6710 | 2937 |
| 5.61 | 889 | 48 |
| 5.72 | 50 | 1 |

calculated using the spin and parity assignments of column B in Table I, while those in column C pertain to column C of Table I.

## COMPARISON WITH EXPERIMENT

The rate of capture to the ground state of $B^{12}$ was calculated by Morita and Fujii.[1] The result here is slightly different, due to the fact that the radial integrals were evaluated to include more significant figures. This is simply the result of a machine calculation and is of no importance, since the relative values of the coupling constants are known to two significant figures at best.

In a recent experiment, E. Maier[5] has measured the capture rates to the individual bound excited states of $B^{12}$. His results, given in Table III, show the percentage of bound-state captures which result in a particular excited state of boron.

The agreement between the measured data and the calculated result for the 0.95-MeV level is good, since the poor energy resolution does not exclude the possibility of gamma cascade from higher levels through the first excited state. Predictions regarding the 1.67-MeV level are in poor agreement with experiment; the lower limit to the calculated result is too large by a factor of 2.3. One possible explanation is that the currently accepted spin and parity assignments of this level are incorrect, and that the transition is actually second forbidden. This would suggest that the 1.67-MeV level may have a spin and parity of $J^\pi = 2^+$. If the experimental data for the 2.62- and 2.72-MeV levels are correct, captures to these states might be expected to be at least second forbidden. Should this be the case, the assignment $J^\pi = 2^+$, $3^+$ is considered reasonable.

TABLE III. Percentage of bound-state captures leading to definite final nuclear states.

| Energy of level above ground state (MeV) | Percentage of bound-state captures: Experiment$^a$ | Theory: Lower limit | Upper limit |
|---|---|---|---|
| 0.95 | 3.83±2.09 | 0.50 | 0.77 |
| 1.67 | 5.20±3.15 | 12 | 24 |
| 2.62 | 1.30±1.90 | 1.4 | 16 |
| 2.72 | | | |

$^a$ Only the 0.95-MeV level is considered a reliable measurement, the other measurements being, at best, upper limits. The experiment did not resolve the 2.62- and 2.72-MeV levels.

Other experiments measuring the rate of capture to all bound states of $B^{12}$ have yielded the following results:

$(9.18\pm0.5)\times10^3$ sec$^{-1}$,[10] $(7.6\pm1.2)\times10^3$ sec$^{-1}$,[12]

$(7.01\pm0.27)\times10^3$ sec$^{-1}$,[11] $(10\ \pm1)\ \times10^3$ sec$^{-1}$.[13]

The results of this calculation establish a lower limit of $8.5\times10^3$ sec$^{-1}$ and an upper limit of $12.7\times10^3$ sec$^{-1}$ for all bound-state captures. This would seem to exclude at least the assignment of $J^\pi = 1^-$ to the 1.67-MeV level. The argument receives some support from the expectation that captures to bound excited states should contribute no more than ten percent to all bound state captures.[13]

Coincidentally, whether doubtful spin and parity are assigned to favor bound or unbound captures, the total absorption rate by $C^{12}$ is predicted to be $1.9\times10^4$ sec$^{-1}$. This is to be compared with the following experimental values:

$(3.73\pm0.11)\times10^4$ sec$^{-1}$,[14] $(3.61\pm0.1)\times10^4$ sec$^{-1}$.[15]

There is recent experimental evidence for virtual states of $B^{12}$ as high as 10 MeV above ground,[16] but the crudity of our nuclear model makes it difficult to judge the extent to which these contribute to the large error.

As to the calculated total capture rate, it is too low by a factor of about 2; the predicted ratio of unbound-to-bound captures has an upper limit of 1.3, while experiment would set a lower limit of about 3.6. Again, the difference is not surprising and rests in part, at least, upon lack of knowledge of highly excited virtual states of $B^{12}$.

## CONCLUSIONS

Although it is practically impossible to include configuration mixing for a nucleus such as $C^{12}$, the calculation could certainly be improved by a more detailed study of the correction to the radial integrals.

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[15] J. L. Lathrop, R. A. Lundy, V. L. Telegedi, R. Winston, and D. D. Yovanovitch, Phys. Rev. Letters **7**, 107 (1961).

[16] R. K. Hobbie, C. W. Lewis, and J. M. Blair, Phys. Rev. **124**, 1506 (1961).

One might investigate the agreement between theory and experiment in beta decays of higher order than allowed, regarding the error as a more correct estimate of the correction necessary for higher order muon captures.

There is still some uncertainty in the experiments which have measured the total absorption rate, so that it is desirable to see performed an experiment which will measure the rate of neutron production directly. This will clearly indicate how much of the error in these calculations is due to the crudity of the approximation.

**ACKNOWLEDGMENT**

The authors wish to express their gratitude to E. Maier for communicating the results of his experiment prior to the appearance of his article.

## APPENDIX

Beginning with Eq. (A8), Morita and Fujii[1] present the reduced matrix elements for the ground-state to ground-state transition. Evaluating all the reduced matrix elements that can occur in a general transition $J_i^{\pi_i}=0^+ \rightarrow J_f^{\pi_f}$, and one obtains the following simplified expressions:

$$[0JJ,\pm]=-\sqrt{3}(2\pi)^{-1}(2j+1)^{1/2}(1Jl|00)W(\tfrac{3}{2}1jl|\tfrac{1}{2}J)\langle R\rangle,$$

$$\langle R\rangle_{0JJ}=\int_0^\infty \phi_l^* e^{-\alpha Z m_\mu' r} j_J(qr)\phi_1 r^2 dr, \tag{A1}$$

$$\langle R\rangle_{0JJ\pm}=\langle R\rangle_{0JJ}\pm[(\alpha Z m_\mu')/q]\int_0^\infty \phi_l^* e^{-\alpha Z m_\mu' r} j_{J\mp 1}(qr)\phi_1 r^2 dr,$$

$$[1\omega J,\pm]=3(2\pi)^{-1}6^{1/2}[(2\omega+1)(2j+1)]^{1/2}(\omega 1 l|00)\langle R\rangle \textstyle\sum_r(2r+1)W(l\omega rJ|11)W(jr\tfrac{3}{2}1|\tfrac{1}{2}J)W(\tfrac{1}{2}1jl|\tfrac{1}{2}r),$$

$$\langle R\rangle_{1\omega J}=\int_0^\infty \phi_l^* e^{-\alpha Z m_\mu' r} j_\omega(qr)\phi_1 r^2 dr, \tag{A2}$$

$$\langle R\rangle_{1\omega J\pm}=\langle R\rangle_{1\omega J}\pm[(\alpha Z m_\mu')/q]\int_0^\infty \phi_l^* e^{-\alpha Z m_\mu' r} j_{\omega\mp 1}(qr)\phi_1 r^2 dr,$$

$$[0JJ\ p]=-(2\pi)^{-1}6^{1/2}(2j+1)^{1/2}\textstyle\sum_\lambda(2\lambda+1)(\lambda Jl|00)\langle R_\lambda\rangle W(\tfrac{3}{2}1\tfrac{1}{2}1|\tfrac{1}{2}\lambda)W(\tfrac{3}{2}\lambda jl|\tfrac{1}{2}J),$$

$$\langle R_\lambda\rangle=\int_0^\infty \phi_l^* e^{-\alpha Z m_\mu' r} j_J(qr)(D_\lambda\phi_1)r^2 dr, \tag{A3}$$

where

$$D_2=\left(\frac{2}{5}\right)^{1/2}\left(\frac{d}{dr}-\frac{1}{r}\right),\quad D_0=-\left(\frac{d}{dr}+\frac{2}{r}\right),$$

$$[1\omega J\ p]=(2\pi)^{-1}\sqrt{3}[(2\omega+1)(2j+1)]^{1/2}\textstyle\sum_\lambda(2\lambda+1)(\lambda\omega l|00)\langle R_\lambda\rangle(-)^{J-\omega}W(lJ\lambda 1|1\omega)W(\tfrac{3}{2}1jl|\tfrac{1}{2}J),$$

$$\langle R_\lambda\rangle=\int_0^\infty \phi_l^* e^{-\alpha Z m_\mu' r} j_\omega(qr)(D_\lambda\phi_1)r^2 dr. \tag{A4}$$